separately, or when the profits have to be distributed according to the values of the policies, or even for the purpose of calculating the surrender values; and should such a table ever be required, I think I have shown that, far from the computations being difficult and laborious, they can be constructed with the greatest ease and accuracy.

The above paper was written before I had seen Dr. Zillmer's ingenious application of M. Thomas' Arithmometer to the construction of Tables of Policy Values. By means of that instrument such tables are more easily constructed and are less liable to error; but without it, his formula must be reduced to that given above before it is applicable for ordinary working.

---

## *Notes on Newton's Formulæ for Interpolation.* By PROFESSOR LUDVIG OPPERMANN, *of Copenhagen.*

### II.

IN the first note I gave the demonstration of Newton's formulæ of Interpolation.

Now the question arises, "How has Newton found this general solution of the problem," or rather, "how has this general form of the problem and the consequent general solution been suggested to Newton?"

The answer is given in the *Methodus Differentialis.*

There Newton proceeds as follows (in the notation I have taken the liberty of making some slight alterations):

*Given,* the $(n+1)$ arguments $a, b, c, d, e, f, g \ldots\ldots$; and the corresponding $(n+1)$ values A, B, C, D, E, F, G . . . .

*Proposed,* to determine the algebraic integer and rational function of the $n$th degree, which assigns the given values to the given arguments.

*Solution.* Let the function in question be denoted by

$$X = \kappa_0 + \kappa_1 x + \kappa_2 x^2 + \kappa_3 x^3 + \ldots\ldots + \kappa_n x^n$$

then the $(n+1)$ quantities $\kappa_0, \kappa_1, \kappa_2, \kappa_3 \ldots\ldots \kappa_n$ are to be determined by $(n+1)$ linear equations of the form

$$\begin{aligned} A &= \kappa_0 + \kappa_1 a + \kappa_2 a^2 + \kappa_3 a^3 + \kappa_4 a^4 + \ldots\ldots \\ B &= \kappa_0 + \kappa_1 b + \kappa_2 b^2 + \kappa_3 b^3 + \kappa_4 b^4 + \ldots\ldots \\ C &= \kappa_0 + \kappa_1 c + \kappa_2 c^2 + \kappa_3 c^3 + \kappa_4 c^4 + \ldots\ldots \\ &\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \end{aligned}$$

By successive eliminations we obtain the following systems of equations:

$$\frac{A-B}{a-b}=\delta'(a,b)=\kappa_1+\kappa_2(a+b)+\kappa_3(a^2+ab+b^2)$$
$$+\kappa_4(a^3+a^2b+ab^2+b^3)+\ldots.$$
$$\frac{B-C}{b-c}=\delta'(b,c)=\kappa_1+\kappa_2(b+c)+\kappa_3(b^2+bc+c^2)$$
$$+\kappa_4(b^3+b^2c+bc^2+c^3)+\ldots.$$
$$\frac{C-D}{c-d}=\delta'(c,d)=\kappa_1+\kappa_2(c+d)+\kappa_3(c^2+cd+d)$$
$$+\kappa_4(c^3+c^2d+cd^2+d^3)+\ldots.$$
$$\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot$$
$$\frac{\delta'(a,b)-\delta'(b,c)}{a-c}=\delta''(a,b,c)=\kappa_2+\kappa_3(a+b+c)$$
$$+\kappa_4(a^2+ab+ac+b^2+bc+c^2)+\ldots.$$
$$\frac{\delta'(b,c)-\delta'(c,d)}{b-d}=\delta''(b,c,d)=\kappa_2+\kappa_3(b+c+d)$$
$$+\kappa_4(b^2+bc+bd+c^2+cd+d^2)+\ldots.$$
$$\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot$$
$$\frac{\delta''(a,b,c)-\delta''(b,c,d)}{a-d}=\delta'''(a,b,c,d)=\kappa_3+\kappa_4(a+b+c+d)+\ldots$$
$$\frac{\delta''(b,c,d)-\delta''(c,d,e)}{b-e}=\delta'''(b,c,d,e)=\kappa_3+\kappa_4(b+c+d+e)+\ldots.$$
$$\frac{\delta'''(a,b,c,d)-\delta'''(b,c,d,e)}{a-e}=\delta^{\text{IV}}(a,b,c,d,e)=\kappa_4+\ldots.$$

&c.

This process must of course end with

$$\delta^n(a,b,c\ldots.)=\kappa_n$$

and then $\kappa_{n-1}$, $\kappa_{n-2}\ldots.\kappa_2$, $\kappa_1$, $\kappa_0$, may be found by substitution from below.

The above is the substance of the two first Propositions in the *Meth. Diff.* In Prop. I. it is asserted that the divisions by which the divided differences are obtained, may always be performed without giving rise to fractional quotients, and this is actually shown as far as the fourth difference. It is to be noted, that in this Proposition the function X is not limited to a finite number of terms. In Prop. II. the number is supposed finite; and it is asserted that the highest divided difference is the coefficient of the

highest term of X, and that from this and the other divided differences we may find the coefficients in the other terms of X, as is actually shown in the example in Prop. I.

Newton then adds simply, *From these Propositions the following may easily be obtained.* ("Ex his Propositionibus quæ sequuntur facile colligi possunt"), and this is perfectly true.

In the two next Propositions are given the formulæ for Interpolation by *central* divided differences, in Prop. III. for equidistant arguments with the constant difference 1, in Prop. IV. for arguments not equidistant. In both Propositions, the two cases of an odd and of an even number of given values, are distinguished.

Then it is shown that the preceding may be applied to *Approximate Interpolation* (Prop. V.) and *Integration* (Prop. VI.) of any function of which a number of values are known.

After these six Propositions comes a Scholium, in which Newton—after pointing out how useful the preceding theory is in the calculation of tables and in solving problems depending on integrations—concludes his important little tract with this theorem: "Through any number of points may be drawn not only a parabolic "curve, but also an infinite number of other curves," or in modern analytical language: "The condition, that to any given number of "arguments ($a$, $b$, $c$ . . . .) correspond as many given values of an "unknown function (A, B, C . . . .), may be satisfied not only by an "integral and rational algebraical function, but also by an infinite "number of other functions." This is actually proved in a very ingenious manner, without making use of periodic functions.

---

*On the Theory of Probabilities. By* Sir John F. W. Herschel, Bart., K.H., M.A., D.C.L., *&c. &c. Being extracts from a review of "Quetelet on Probabilities," which appeared in the "Edinburgh Review" for July,* 1850.*

[Reprinted by permission of the author and publishers.]

THE theory of Probabilities has been characterized by Laplace, one of those who have contributed most largely to its advance,— as "good sense reduced to a system of calculation;" and such, no

* This essay contains so much that is of permanent interest to all students of the Theory of Probabilities that we believe we are doing our readers a real service in reproducing the greater part of it. We would gladly have reprinted the whole, but for the limited amount of space at our disposal; but those who may wish to read the essay in its entirety, will find it in the collected volume of Sir John Herschel's essays published by Messrs. Longmans in 1857.—Ed. *J. I. A.*